



Name:

model answer

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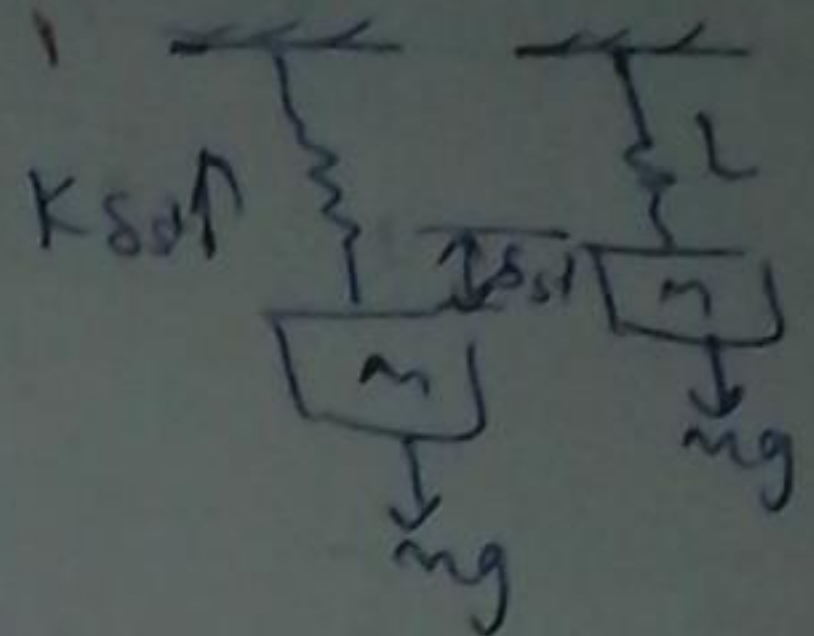
Answer the following Questions:

1) a) How can you find the natural frequency of a system by measuring its static deflection? [3 Marks]

Solution

$$\delta_{st} = \text{stat. deflection} \Rightarrow K \delta_{st} = mg$$

$$\frac{K}{m} = \frac{g}{\delta_{st}} = \omega_n^2 \Rightarrow \omega_n = \sqrt{\frac{g}{\delta_{st}}}$$



b) For a vibrating single degree of freedom system $m=10\text{kg}$, $K=250\text{N/m}$, and $C=45\text{ N.s/m}$. If the initial displacement and velocity of the mass are 15 mm and 5 mm/s , Find the solution representing the motion of the mass. (7 marks)

Solution

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{250}{10}} = 5 \text{ rad/sec}$$

$$m\ddot{x} + c\dot{x} + Kx = 0$$

$$\zeta = \frac{c}{c_c} = \frac{45}{2 \times 10 \times 5} = 0.45 < 1 \Rightarrow \text{underdamped}$$

$$\omega_d = \sqrt{1 - \zeta^2} \omega_n = \sqrt{1 - (0.45)^2} \times 5 = 4.47 \text{ rad/sec}$$

$$x(t) = e^{-\zeta \omega_n t} \left[x_0 \cos \omega_d t + \frac{\dot{x}_0 + \zeta \omega_n x_0}{\omega_d} \sin \omega_d t \right]$$

$$x(t) = e^{-2.25t} [0.015 \cos 4.47t + 0.009 \sin 4.47t]$$